

Black hole masking and black hole thermodynamics

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Masking of black holes means that, for given total mass and Hawking temperatures, these data may correspond to either "pure" black hole or a black hole of a lesser mass surrounded by a massive shell. It is shown that there is one-to one correspondence between this phenomenon and thermodynamics of a black hole in a finite size cavity: masking of black holes is possible if and only if there exists at least one locally unstable black hole solution in the corresponding canonical ensemble.

PACS numbers: 04.70Bw, 04.20.Gz, 04.40 Nr

I. INTRODUCTION

In recent years, the phenomenon of mimicking black holes attracts the particular attention. It implies that, even with reliable observational data at hand, two quite different types of objects are compatible with a given set of data. An observer should make a choice between a true black hole or a compact body with a size slightly bigger than the gravitational radius [1] (see also [2] and references cited there). For a remote observer, they reveal themselves almost indistinguishable gravitationally, although in the vicinity of a (quasi)horizon the difference becomes crucial. Meanwhile, there is also another phenomenon when one is led to choose not between a black hole and its mimicker but between different types of black hole configurations. Namely, as was shown in [3], the measurement of the total mass and (supposing that an observer can measure such things in principle) Hawking temperature leaves an uncertainty. It is impossible to learn whether one deals with a "pure" black hole or a black hole of a lesser mass plus a massive shell. Such a phenomenon was called in [3] "masking". It is just phenomenon which will be discussed below.

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The aim of the present Letter is to draw attention that, actually, there is close correspondence between two seemingly different areas - masking black holes from the viewpoint of a distant observer and thermodynamics of black holes enclosed in a finite size cavity. Although our system corresponds to the microcanonical ensemble (the total mass is fixed, while the Hawking temperature for a given mass can be calculated), the phenomenon of masking has roots in the properties of the canonical ensemble. Thus, (i) two quite different phenomenon (black hole masking and finite size thermodynamics of black holes) turn out to be mutually connected, (ii) the complementarity between two quite different types of the gravitational ensemble reveals itself.

II. BASIC FORMULAS

Consider a shell around a spherically symmetric black hole, so that the metric reads

$$ds^2 = -A^2 U_1 dt^2 + \frac{dr^2}{V_1} + r^2 d\omega^2, \quad r_+ \leq r \leq R, \quad (1)$$

$$ds^2 = -U_2 dt^2 + \frac{dr^2}{V_2} + r^2 d\omega^2, \quad r \geq R. \quad (2)$$

We do not specify the explicit form of the metric. We only assume that r_+ is the horizon, so $U_1(r_+) = 0 = V_1(r_+)$. In particular, it can be the Schwarzschild, Reissner-Nördstrom or Scharazchild - de Sitter metric, etc. We also suppose that inside and outside the type of matter or fields is the same, so the metric functions differ by the constants of integration only:

$$U_1 = U(r, m_1), \quad U_2 = U(r, m_2), \quad V_1 = V(r, m_1), \quad V_2 = V(r, m_2). \quad (3)$$

Here, $m_1 = \frac{r_+}{2}$, m_2 is the total ADM mass of the system,

$$m_2 = m_1 + m_s, \quad (4)$$

m_s is the contribution to the ADM mass from the shell. As we consider static systems, we assume that $R > 2m_2$. The metric can depend on other independent parameters which are supposed to be the same inside and outside. For brevity, we omit such a dependence in formulas.

In the state of thermal equilibrium, the system is characterized by the constant temperature parameter T_0 having the meaning of the temperature measured by an observer at

infinity. On the shell, the local Tolman temperature

$$T = \frac{T_0}{\sqrt{U_2(R)}}. \quad (5)$$

The continuity of the metric induced on $r = R$ requires that

$$A = \sqrt{\frac{U_2(R)}{U_1(R)}}. \quad (6)$$

Then, the temperature T_0 is equal to [4], [3]

$$T_0 = AT_H^{(1)} \quad (7)$$

where

$$T_H^{(1)} = \frac{\sqrt{U_1'(r_+)V_1'(r_+)}}{4\pi} = T_H(m_1) \quad (8)$$

is the Hawking temperature calculated for the black hole metric which is obtained from (1) by omitting the screening factor A . It follows from (6) - (7) that

$$T = \frac{T_H^{(1)}}{\sqrt{U_1(R)}}. \quad (9)$$

Eqs. (5), (9) express the condition of the thermal equilibrium: local temperatures calculated from both sides of the shell coincide.

III. MASKING AND INTERPLAY BETWEEN MICROCANONICAL AND CANONICAL ENSEMBLES

Up to now, we simply used general formulas which follow from thermodynamics and the conditions of matching two metrics. The phenomenon of masking arises if, additionally, the temperature at infinity can be interpreted as the Hawking temperature of a black hole without a shell but having the same total mass:

$$T_0 = T_H(m_2) \quad (10)$$

where m_2 is its ADM mass coinciding with (4).

For simplicity, we assume that the shell does not carry an electric charge and is characterized by the radius and mass only. Then, it follows from (5) and (10) that

$$T = f(m_1, R) = f(m_2, R) \quad (11)$$

where $f(m, R) = \frac{T_H(m)}{\sqrt{U(R, m)}}$.

In general, we can suppose that eq. (11) has a set of roots m_i where $i = 1, 2, \dots, N$.

Now, the key observation consists in that eq. (11) arises in the canonical ensemble. Namely, if we consider a cavity of the areal radius R and fix the local temperature T on its boundary, the equation

$$T = f(R, m) \quad (12)$$

determines masses of a black hole which can exist inside [5]. For the Schwarzschild case, if the temperature is high enough, there are two roots, one unstable m_1 and the other - stable with $m_2 > m_1$ (see [5] for details and discussion below). In general, there exists a number of roots m_i , $i = 1, 2, \dots, N$. In what follows we shall also make a physically reasonable assumption that the energy is a monotonically increasing function of the ADM mass, $\frac{dE}{dm} > 0$. Then, we have the following

Theorem. Masking of black holes is possible if and only if there exists at least one locally unstable black hole solution in the corresponding canonical ensemble.

Proof. (i) Let us suppose that masking is possible and there are no unstable roots. This means that the heat capacity $C = \frac{dE}{dT} > 0$. As $\frac{dE}{dm} > 0$, we see that $\frac{dm}{dT} > 0$, so $f(m, R)$ is the monotonic function of m . Hence, eq. (12) has only one root and masking is impossible, so we obtained the contradiction. Thus, if there is masking, unstable roots are inevitable.

(ii) In a similar manner, one can prove that the reverse is also true. Let locally unstable roots do exist. Then, the function $f(m)$ has at least one segment with $\frac{\partial f}{\partial m} < 0$. From the other hand, for R close to r_+ , the hole occupies almost all the cavity, $U \rightarrow 0$, $f \rightarrow \infty$, so the function f increases when r_+ approaches R . In doing so, the contribution to the mass from the matter between the gravitational radius and the boundary is negligible, so $m \approx \frac{r_+}{2} \rightarrow \frac{R}{2}$. Thus, there exist the branch with $\frac{\partial f}{\partial m} > 0$. Between both branches there is at least one local minimum. Taking the value of T bigger than this minimum (but smaller than the maximum value of f nearest to it from the left if such a maximum exist), we obtain the solution with at least two different roots, so that masking is indeed possible. Thus, the allowed range of solutions falls into the interval determined by the inequality

$$f(m, R) \geq f_0(R) \quad (13)$$

where $f_0(R) = f(m_0(R), R)$ is the minimum of f , $(\frac{\partial f}{\partial m})_{m=m_0(R)} = 0$.

It is worth stressing that the correspondence which we established relates two different types of systems. Type 1 implies that the space-time is asymptotically flat, with the total mass m_2 fixed that corresponds to the *microcanonical* ensemble. Type 2 represents the *canonical* ensemble where the physical manifold is restricted by inequality $r \leq R$ that represents the interior of the finite size cavity, but there is no an external remote observer and there is no shell at all. In a sense, we have a complementarity of two ensembles and of two types of boundaries (the thing shell in an infinite space and the boundary enclosing the physical manifold).

IV. EXAMPLE: SCHWARZSCHILD BLACK HOLE

Now I illustrate the above consideration using the Schwarzschild black hole as an example. The possibility of masking such a black hole was pointed out in [3], thermodynamics of finite size system for such black holes was considered in [5] but now my aim is to compare both phenomena. Now, in geometric units $T_H = (8\pi m)^{-1}$, $U = V = 1 - \frac{2m}{r}$, $f = 8\pi m \sqrt{1 - \frac{2m}{R}}$, $E = R(1 - \sqrt{V(R)})$. The condition of masking (10), (11) can be written in the form

$$\frac{1}{8\pi m_2 \sqrt{1 - \frac{2m_2}{R}}} = \frac{1}{8\pi m_1} \frac{1}{\sqrt{1 - \frac{2m_1}{R}}} \quad (14)$$

which coincides with eq. (3.2) of [3]. This exactly corresponds also to the equation that, for a fixed local temperature T on the boundary, defines possible values of the black hole mass inside the cavity:

$$T = f(m, R) = (8\pi m \sqrt{1 - \frac{2m}{R}})^{-1}. \quad (15)$$

For a given R , the quantity $f(m, R)$ as the function of m (or the gravitational radius $r_+ = 2m$) has two branches - monotonically decreasing and monotonically increasing ones which meet in the point of the minimum of f where $m = \frac{R}{3}$. For given T, R the solutions of this equation exist for $RT > \frac{\sqrt{27}}{8\pi}$. There are two roots with masses $m_1 < m_2$, the light one on the decreasing branch of f is locally unstable, the heavy one on the increasing branch of f is locally stable (see [5] for details).

To render the aforementioned inequality in terms of mass (thus translating the properties of the canonical ensemble to those of the microcanonical one), it is necessary to find the minimum of $f(m, R)$ with respect to m for a given R . The solution lies above such a

minimum. Then, it is easy to show that $R < 3m_2$. In combination with the condition $R > 2m_2$, we obtain the restriction

$$2m_2 < R < 3m_2. \quad (16)$$

This equation is a particular case of eq. (13), now $f_0 = \frac{3\sqrt{3}}{8\pi R}$. Thus, any "pure" black hole with a given mass m_2 has an infinite set of "doubles" obtained with the help of the shell of different radii in the interval (16). Vice versa, if from the very beginning we take a black hole surrounded by a shell of the areal radius R , such a configuration has a pure black hole as its double. The total mass should lie in the interval $\frac{R}{3} < m_2 < \frac{R}{2}$. If the masses of the black hole and shell are also fixed, not any configuration can be masked since for $m_2 > \frac{R}{3}$ inequality (16) is violated. If it is satisfied, an observer at infinity cannot distinguish between two configurations - the black hole without the shell having the mass m_2 and the black hole with the mass m_1 surrounded by the shell of the mass $m_s = m_2 - m_1$. Both configurations have the same total ADM mass m_2 and the same Hawking temperature measured at infinity $T_H = \frac{1}{8\pi m_2}$.

If one places the shell at $R = 3m_2(1 - \delta)$, $\delta \ll 1$, it follows from (14) that $m_1 \approx \frac{R}{3}(1 - \delta)$, $m_s \approx \frac{2R}{3}\delta$, so in the limit $\delta \rightarrow 0$ the effect of masking almost vanishes. If one considers the shell at $R = 2m_2(1 + \varepsilon)$, $\varepsilon \ll 1$, it turns out that $m_1 \approx \frac{1}{8\pi T} \approx m_2\sqrt{\varepsilon} \ll m_2$, $m_s \approx m_2$. Thus, the mass stems almost entirely from the shell, whereas the contribution of the black hole inside the shell becomes negligible. In this respect, such a situation is close to that for black hole mimickers where there is no a black hole at all, the size of the body approaching the gravitational radius. In doing so, large tangential stresses develop on the shell to maintain it in equilibrium [2] but they are irrelevant for a distant observer.

V. SUMMARY

In general, the canonical ensemble implies that the system is enclosed inside some cavity of the finite size, so the region with $r > R$ is not part of the physical system at all. By contrary, the phenomenon of masking implies that measurement are done at infinity. Nonetheless, it turned out that these so different (in a sense, mutually complimentary) phenomena are intimately tied. The effects of finite size in black hole thermodynamics [5] are sometimes considered as a pure academic matter having no observational consequences. However,

if we assume that the Hawking radiation is detectable, properties of finite size black hole thermodynamics should be taken into account just in observations to single out the potential effect of black hole masking.

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